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Omnidirectional total reflection for liquid surface waves propagating over a bottom with one-dimensional periodic undulations

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We study theoretically the propagation of liquid surface waves over a bottom with one-dimensional (1D) periodic undulations. We find a general criterion for omnidirectional total reflection in such a system. Numerical simulations based on a transfer matrix method demonstrate unambiguously the existence of omnidirectional total reflection for liquid surface waves propagating over a bottom with 1D periodic undulations.

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The interactions of surface water waves with uneven bottoms are a fundamental hydrodynamics problem [1,2], which has important scientific values and potential applications such as coastal engineering. The scattering of water waves by a finite number of one-dimensional (1D) periodically modulated ripples or bars on an horizontal, flat bottom has received considerable attention for many years [1–6]. Many interesting phenomena were found such as Bragg resonance, a terminology from solid state physics [7]. As we know from solid state physics, Bragg resonance leads to complicated band structures for classical waves propagating in periodic structures. Between bands there may exist a band gap for waves with frequency within which propagation is absolutely forbidden. The idea of band structures and band gaps has been extended to photonic crystals for electromagnetic waves [8,9], sonic crystals for elastic waves [10], and even to the propagation of liquid surface waves in periodic structures [11–18]. From band structures we can get deeper insight into the propagation of liquid surface waves in periodic structures.

It was known that there does not exist a complete band gap (along all directions) in infinite 1D photonic crystals, e.g., two dielectric layers stacking alternatively. However, it was shown recently that a finite 1D photonic crystal can totally reflect incident light over a certain frequency range at all angles, i.e., omnidirectional total reflection [19–22]. The central idea resides in that if there are no propagating modes that can couple an incident wave of any angle, omnidirectional total reflection can occur. Omnidirectional total reflection in 1D photonic crystals is of great scientific and practical significance [23].

In the present work, we show theoretically the analog of omnidirectional total reflection in liquid surface waves propagating over a bottom with a 1D periodic undulation. The schematic view of the bottom structure studied is shown in Fig. 1. The undulated structure can be obtained as follows. A flat board of infinite length along the y direction is put on the bottom. An array of solid bars with rectangular cross section is then placed on the board periodically along the x direction. The liquid depth over the bottom, the board, and bars is denoted by h_0, h_1, and h_2, respectively. The spacing between adjacent bars is d_1 and the horizontal dimension of bars is d_2. The calm liquid surface is set at . Thus, the periodically arranged bars serve as a 1D periodically undulated structure.

For incompressible inviscid liquids, the governing equations within the linear wave theory are given by [2]

$$\nabla^2\Phi + k^2\frac{\partial^2\Phi}{\partial z^2} = 0 \text{ for } -h \leqslant z \leqslant 0, \tag{1a}$$

$$\Phi + \frac{g}{\omega^2}\frac{\partial\Phi}{\partial z} = 0 \text{ for } z = 0, \tag{1b}$$

$$\frac{\partial\Phi}{\partial z} = 0 \text{ for } z = -h, \tag{1c}$$

where Φ is the velocity potential, ω is the angular frequency, k is the wave vector, and g is the gravitational acceleration. At the boundary between two different regions, we have the following matching conditions:

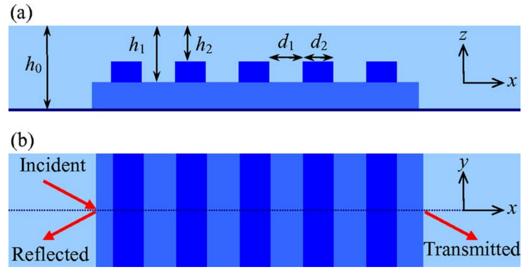

FIG. 1. (Color online) Schematic (a) side and (b) top views of a bottom with a 1D periodic undulation.

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$$\Phi_1 = \Phi_2, \tag{2a}$$

$$\frac{\partial \Phi_1}{\partial x} = \frac{\partial \Phi_2}{\partial x} \text{ for } -h_1 \leqslant z \leqslant 0, \tag{2b}$$

$$\frac{\partial \Phi_2}{\partial x} = 0 \quad \text{for } -h_2 \leqslant z \leqslant -h_1. \tag{2c}$$

In the system under study both the traveling wave modes and the evanescent wave modes are present. The complete solutions of the velocity potentials in different regions have the similar form, given by [2,24]

$$\Phi_m(\mathbf{r},z) = (A_m e^{i\mathbf{k}\cdot\mathbf{r}} + B_m e^{-i\mathbf{k}\cdot\mathbf{r}})X_m(z) + \sum_{s=1}^{\infty} c_m(k_s) e^{\mathbf{k}_s\cdot\mathbf{r}} f_m(z,k_s), \tag{3}$$

where $\mathbf{r}=(x,y)$, m is the index standing for different regions, k and k_s are the wave vectors for the traveling and evanescent waves, respectively, and $X(z)$ and $f(z,k_s)$ are the vertical component of the traveling wave modes and the evanescent wave modes, respectively, given by

$$X(z) = 2\sqrt{k}\frac{\cosh[k(z+h)]}{\sqrt{\sinh(2kh)+2kh}}, \tag{4}$$

$$f(z,k_s) = 2\sqrt{k_s}\frac{\cos[k_s(z+h)]}{\sqrt{\sin(2k_s h)+2k_s h}}. \tag{5}$$

The coefficients A, B, C in different regions should be related by proper matching conditions of Eqs. (2), as done in Ref. [2]. The dispersion relation for the traveling wave modes follows

$$\omega^2 = gk\left(1 + \frac{k^2 T}{\rho g}\right)\tanh(kh), \tag{6}$$

while for the evanescent wave modes it is given by the

$$\omega^2 = -gk_s\left(1 + \frac{k_s^2 T}{\rho g}\right)\tan(k_s h), \quad s = 1,2,3\ldots, \tag{7}$$

where T is the liquid surface tension and ρ is the liquid density.

A transfer matrix method which is inclusive of the evanescent waves is extended to the study of liquid surface waves propagating over uneven bottoms. This method has been frequently used in the calculations of optical properties of multiple thin films [25,26]. Within the framework of the transfer matrix method, the band structures, transmission, and reflection can be calculated. The liquid used in our calculations is the same as in our previous experiments to observe superlensing, self-collimation phenomena, and band gaps in liquid surface waves [16–18]. The reason for using this liquid is due to the fact that we can carry out experiments along the setups established [16–18]. It should be noted that our methodology and results can be extended to water waves in a rather straightforward way.

Figure 2 shows the calculated band structure of liquid surface waves over a bottom with an infinite 1D periodic undulation for the propagating direction along x. Similar to that in solids and photonic crystals, wave propagation in periodic structures is characterized by complicated band structures. Between bands there exist a series of band gaps owing to Bragg resonance. For waves with frequency located into a band, propagation is allowed. On the contrary, propagation is forbidden for waves with frequency located into a band gap. It can be found the width of band gaps at low frequencies is larger in general than that at high frequencies. This is due to the fact that low frequency waves sense the undulation stronger than high frequency waves.

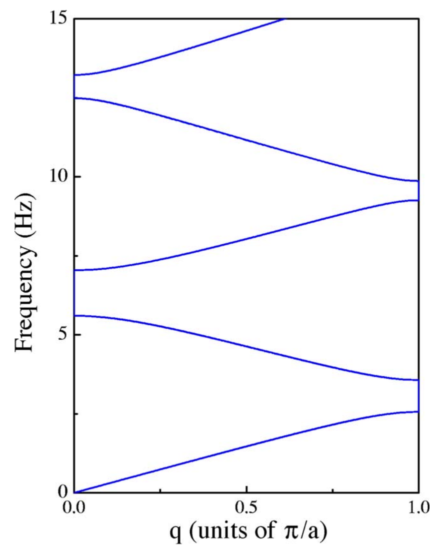

FIG. 2. (Color online). Calculated band structure of liquid surface waves over a bottom with an infinite 1D periodic undulation for the Bloch wave vector along x. Structural parameters are $d_1=d_2=10$ mm, $h_1=5$ mm, and $h_2=1$ mm. The Bloch wave vector q is in units of π/a, where $a=d_1+d_2$ is the lattice constant of the 1D undulation.

For an arbitrary propagating direction, the structure with an infinite 1D undulation is periodic along the x direction. But it is homogeneous along the y direction. Thus, for an arbitrary direction of propagation, it is better to examine the projected band structure over the tangential component q_y of the Bloch wave vector $\mathbf{q}$, shown in Fig. 3. Since along the x direction the structure is periodic, the Bloch wave vector component q_x should be restricted to the range $-\pi/a \leqslant q_x \leqslant \pi/a$. The allowed mode frequencies $\omega_n(\mathbf{q})$ for each choice of $\mathbf{q}$ constitute the projected band structure, where n is the band index.

For a wave of arbitrary propagating direction over a bottom, its angular frequency $\omega(k_x,k_y)$ is given by the dispersion relation of Eq. (6), where $k=(k_x^2+k_y^2)^{1/2}$, so in general $\omega(0,k_y) \leqslant \omega(k_x,k_y)$ since $k_y \leqslant k$. The line $\omega(0,k_y=k)$ thus can be viewed as a *wave line*, similar to the light line for electromagnetic waves [19] under which there are no propagative modes. Wave lines for different liquid depth h_0 are also plotted in Fig. 3.

For waves incident from a region with liquid depth h_0 upon a structure with a finite 1D undulation, the system is no longer infinitely periodic along the x direction. However, q_y is still a valid symmetrical label since along the y direction

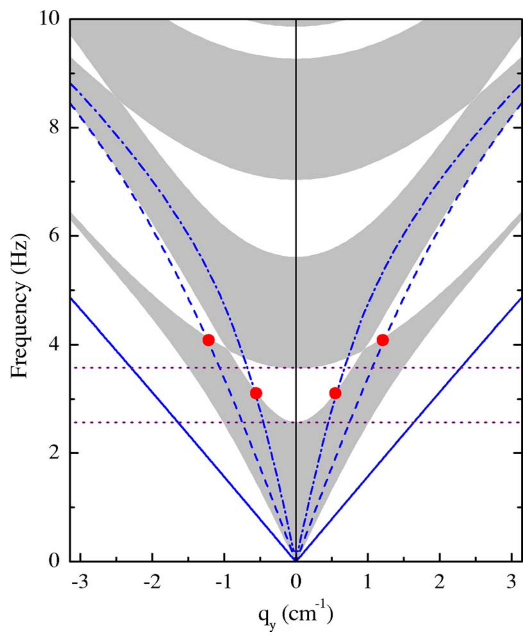

FIG. 3. (Color online) Projected band structure of the same structure in Fig. 2 for arbitrary directions of propagation over the tangential Bloch wave vector q_y. Shaded and white regions represent bands and band gaps, respectively. Solid, dashed, and dash-dotted lines stand for the wave lines for $h_0=1,5,15$ mm, respectively. Two horizontal dotted lines denote the edges of the first band gap at $q_y=0$. Black dots are the crossing points between wave lines and the upper edge of the first band.

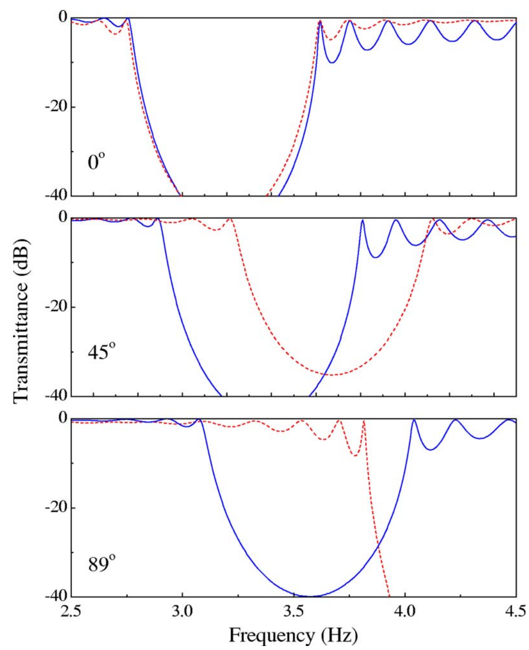

FIG. 4. (Color online) Calculated transmittance spectra of a structure with a 1D undulation of 30 periods at various angles of incidence. Structural parameters are the same as in Fig. 2. Solid and dashed lines stand for $h_0=15$ and 5 mm, respectively.

the system is infinitely homogeneous. Thus, the projected band structure can still be a good reference for discussing transmissive properties. In the following discussions, we focus on the first band gap although higher-order band gaps can be discussed similarly. The first band gap at $q_y=0$ ranges from 2.56 to 3.57 Hz, defined by the upper and lower edges of the first and second bands, respectively. It is interesting to note that the first and second bands cross at about $q_y=\pm 0.95$ mm^{-1}, corresponding to the internal Brewster's angle between two media with liquid depth of h_1 and h_2. The existence of the Brewester's angle in liquid surface waves can be understood by the similar physical nature between liquid surface waves and p-polarized electromagnetic waves [27,28]. Physically, the boundary conditions satisfied for liquid surface waves at the interface between two media are rather similar to those for p-polarized electromagnetic waves. Wave lines may cross the upper edge of the first band at certain q_y, denoted by dots in Fig. 3. If these crossing points are above the upper edge of the first band gap at $q_y=0$, omnidirectional total reflection cannot exist. Thus, the criterion for omnidirectional total reflection is that the crossing points between the wave lines and the upper edge of the first band are lower than the upper edge of the first band gap at $q_y=0$. This criterion is also valid for higher-order band gaps.

Using the criterion discussed above, we can determine the critical liquid depth h_0 that enables omnidirectional total reflection, being 8.2 mm. For h_0 smaller than this critical depth, omnidirectional total reflection cannot occur. It can be found from Fig. 3 that for $h_0=1$ or 5 mm, no omnidirectional total reflection is expected since there is no crossing or the crossing points between the wave lines and the upper edge of the first band are above the upper edge of the first band gap at $q_y=0$. For $h_0=15$ mm, however, the crossing points are lower than the upper edge of the first band gap at $q_y=0$. Consequently, within the frequency range between the crossing points and the upper edge of the first band gap at $q_y=0$, omnidirectional total reflection is expected.

To confirm that omnidirectional total reflection does exist, we need to calculate the transmittance spectra for a bottom with a finite 1D periodic undulation at various angles of incidence, shown in Fig. 4. For $h_0=5$ mm, the total reflection frequency range at 0° and 45° incidence overlaps. However, at 89° incidence, waves can transmit for frequencies within the above overlapping frequency range. Thus, there does not exist omnidirectional total reflection. For $h_0=15$ mm, over the frequency range approximately from 3.16 to 3.57 Hz, waves are totally reflected even at 89° incidence, indicating the existence of omnidirectional total reflection. Our proposal that renders omnidirectional total reflection may have practical applications in ocean engineering. But one must consider that there is a broad range of incoming frequencies. This can be achieved by the proper choice of the structural parameters or by stacking different undulatory structures to enlarge the frequency range of omnidirectional total reflection [22].

In summary, based on a transfer matrix method, we studied theoretically the band structures and transmissive properties of liquid surface waves propagating over a bottom with a 1D periodic undulation. We found that omnidirectional total reflection can exist in this system if the undulatory parameters are properly chosen. A general criterion that enables omnidirectional total reflection was proposed. The existence of omnidirectional total reflection for liquid surface waves propagating over a bottom with 1D periodic undulations may manifest potential applications.

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